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2. Two smooth uniform spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg . The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1 \text{ m s}^{-1}$ and the velocity of T is $\mathbf{u}_2 \text{ m s}^{-1}$. The coefficient of restitution between the spheres is 0.5 . Immediately after the collision the velocity of S is $(-\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$ and the velocity of T is $(\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$. Given that when the spheres collide the line joining their centres is parallel to \mathbf{i} ,

(a) find

(i) \mathbf{u}_1 ,

(ii) \mathbf{u}_2 .

(6)

After the collision, T goes on to collide with a smooth vertical wall which is parallel to \mathbf{j} . Given that the coefficient of restitution between T and the wall is also 0.5 , find

(b) the angle through which the direction of motion of T is deflected as a result of the collision with the wall,

(5)

(c) the loss in kinetic energy of T caused by the collision with the wall.

(3)



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3. At 12 noon, ship *A* is 8 km due west of ship *B*. Ship *A* is moving due north at a constant speed of 10 km h^{-1} . Ship *B* is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to *A* as possible.
- (a) Find the bearing on which ship *B* moves. (4)
 - (b) Find the shortest distance between the two ships. (3)
 - (c) Find the time when the two ships are closest. (3)



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4. A particle of mass m is projected vertically upwards, at time $t = 0$, with speed U . The particle is subject to air resistance of magnitude $\frac{mgv^2}{k^2}$, where v is the speed of the particle at time t and k is a positive constant.

(a) Show that the particle reaches its greatest height above the point of projection at time

$$\frac{k}{g} \tan^{-1} \left(\frac{U}{k} \right). \quad (6)$$

(b) Find the greatest height above the point of projection attained by the particle. (6)



5.

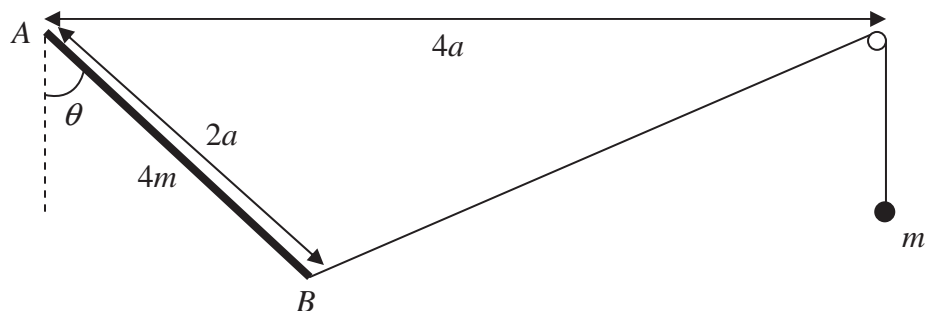


Figure 1

The end A of a uniform rod AB , of length $2a$ and mass $4m$, is smoothly hinged to a fixed point. The end B is attached to one end of a light inextensible string which passes over a small smooth pulley, fixed at the same level as A . The distance from A to the pulley is $4a$. The other end of the string carries a particle of mass m which hangs freely, vertically below the pulley, with the string taut. The angle between the rod and the downward vertical is θ , where $0 < \theta < \frac{\pi}{2}$, as shown in Figure 1.

(a) Show that the potential energy of the system is

$$2mga(\sqrt{5-4\sin\theta}-2\cos\theta)+\text{constant}.$$
(5)

(b) Hence, or otherwise, show that any value of θ which corresponds to a position of equilibrium of the system satisfies the equation

$$4\sin^3\theta-6\sin^2\theta+1=0.$$
(5)

(c) Given that $\theta = \frac{\pi}{6}$ corresponds to a position of equilibrium, determine its stability.

(5)



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Question 5 continued

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6. Two points A and B lie on a smooth horizontal table with $AB = 4a$. One end of a light elastic spring, of natural length a and modulus of elasticity $2mg$, is attached to A . The other end of the spring is attached to a particle P of mass m . Another light elastic spring, of natural length a and modulus of elasticity mg , has one end attached to B and the other end attached to P . The particle P is on the table at rest and in equilibrium.

(a) Show that $AP = \frac{5a}{3}$. (4)

The particle P is now moved along the table from its equilibrium position through a distance $0.5a$ towards B and released from rest at time $t = 0$. At time t , P is moving with speed v and has displacement x from its equilibrium position. There is a resistance to motion of magnitude $4m\omega v$ where $\omega = \sqrt{\left(\frac{g}{a}\right)}$.

(b) Show that $\frac{d^2x}{dt^2} + 4\omega \frac{dx}{dt} + 3\omega^2 x = 0$. (5)

(c) Find the velocity, $\frac{dx}{dt}$, of P in terms of a, ω and t . (8)



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Question 6 continued

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